MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

4. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania

I have two circular grindstones, each ½ in. thick. One is 6 in. and the other 4½ in. in diameter, the aperture at center of each being 1½ in. If when in motion they are continually tangent to each other, and ½ cu. in. is ground off the larger wheel and ¼ cu. in. off the smaller in the first hour, how must their speed be increased so that the same amount per hour may be ground off each wheel until one is worn out? If in the first hour the larger wheel makes a revolutions, and the smaller b, how many must each make in each succeeding hour?

I. Solution by ALFRED HUME, C. E., Professor of Mathematics, University of Mississppi, University P. O., Mississippi.

It is assumed that the stones have independent motions, and that the diminution of their diameters caused by the mutual sliding friction is directly proportional to the number of revolutions. Let $ABCD$ be a rectangle which, revolving about CD, generates the larger grindstone. Let CE be the stone's radius at the end of the first hour. The volume ground off is, by the Theorem of Pappus,

$$2\pi\left(CA-\frac{AE}{2}\right)(AB\times AE);$$ or, substituting the given values, $CA=3$, $AB=\frac{1}{2}$, and the volume being ½ cu in., AE is found to be $3-\sqrt{9-\frac{1}{\pi}}$. At the end of the first hour, then, the radius of the stone, CE, is $\sqrt{9-\frac{1}{\pi}}$. In the same way it may be shown that at the end of the second hour, another ½ cu. in. having been ground off, the radius is $\sqrt{9-\frac{2}{\pi}}$.

At the end of the third hour, the radius is $\sqrt{9-\frac{3}{\pi}}$.

At the end of the nth hour, the radius is $\sqrt{9-\frac{n}{\pi}}$.

If the stone makes a revolutions during the first hour, during the second it must make $\dfrac{\sqrt{9-\frac{1}{\pi}}-\sqrt{9-\frac{2}{\pi}}}{3-\sqrt{9-\frac{1}{\pi}}}a$, and during the third,

$$\frac{\sqrt{9-\frac{2}{\pi}}-\sqrt{9-\frac{3}{\pi}}}{3-\sqrt{9-\frac{1}{\pi}}}a.$$

The revolutions made in the second, 3rd, and nth hours are, therefore,

$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-1}-\sqrt{9\pi-2})\,a,$$
$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-2}-\sqrt{9\pi-3})\,a,$$
$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-n+1}-\sqrt{9\pi-n})\,a, \text{ respectively.}$$

The time required to grind away all of the stone is given by the equation,

$\sqrt{9-\frac{n}{\pi}}=\frac{3}{4}$, from which $n=\frac{135\pi}{16}$, the number of hours.

Similarly, the number of revolutions made by the smaller stone during the nth hour is found to be $\frac{1}{8}(\sqrt{81\pi}+\sqrt{81\pi-8})(\sqrt{81\pi-8(n-1)}-\sqrt{81\pi-8n})$ b, and the time required to grind it away, $\frac{69\pi}{8}$. Therefore the larger wears out first, and at this time the smaller is a cylindrical shell whose thickness is $\frac{3}{4}(\sqrt{\frac{3}{2}}-1)$ inches.

This problem was also solved by *P. H. PHILBRICK*, and *H. W. DRAUGHON.*

5. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A cubic mile of saturated air at 18°C. is cooled to a temperature of 10°C. How many tons of rain will fall?

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

According to Silliman's Table the weight of the aqueous vapor in a cubic foot of saturated air at 18°C., $=64\frac{2}{5}$°F., $=6.663$ avoirdupois grains; and the weight of that in a cubic foot of the same kind of air at 10°C., $=50$°F., $=4.089$ avoirdupois grains. The difference of these weights is the weight of the rain that will fall from a cubic foot of air. Hence the weight of the rain that will fall from a cubic mile of air is

$$R=\frac{(5280)^3\times 1287}{7000\times 2000\times 500}=27125.776 \text{ tons.}$$

Also solved by the *PROPOSER.*

6. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Two men wish to buy a grindstone 42 inches in diameter and one foot thick at the center. To what thickness at the outer edge should the stone uniformly taper from the center so that each man may grind off 18 inches of the diameter and both have equal shares, the central six inches of the diameter being waste?

I. Solution by Professor P. H. PHILBRICK, M. S., C. E, Lake Charles, Louisiana; and A. L. FOOTE, No. 80 Broad Street, New York City.

Let $ABHG$ represent a half section of the stone through the centre. Draw the centre line $KLMN$; make $KL=3$, $LM=MN=9$ inches; and draw CD and EF parallel to AB. Let $GH=x$, and let G and g be the centers of gravity of $CDEF$ and $EFGH$. It is easy to show that,